

# Path optimization for the LPBF additive manufacturing process (LPBF)

PhD funded by SOFIA project

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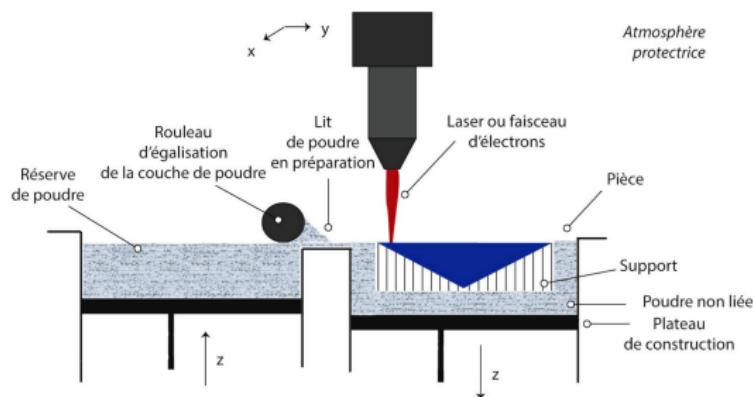
21 Mai 2019



# Introduction



# The LPBF process



Process description

(<http://ceal-aluquebec.com/fabrication-additive>)

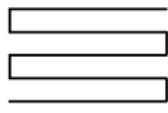
■ **Advantages :**  
Freedom in the design.

■ **Drawbacks :**  
quality of the final object,  
related to the manufacturing  
process (powder, thermal,  
mechanical phenomena)

# Laser path

## State of the art :

parallel



contour



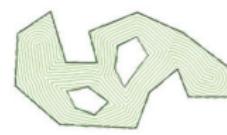
spiral



continuous



Medial Axis Transformation



- Genetic algorithm.

**Goal : path optimization without basing it upon any pattern.**

# Outline

**1** Introduction

**2** Model

**3** Steady problem

**4** Unsteady problem

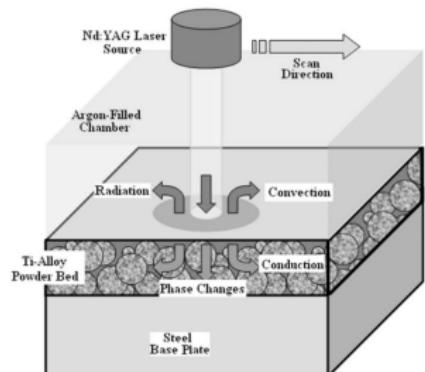
**5** Conclusion and perspectives



# Model



# LPBF modelization



LPBF process  
(Roberts et al. [7])

## Microscopic scale

(Megahed et al. [6] and DebRoy et al. [3])

- accurate model for the change of state, melting pool,
- 4 states considered: powder, solid, liquid, gaz.

## Macroscopic scale

(Allaire and Jakabčin [1], Van Belle [9], and Megahed et al. [6])

- conduction, convection and radiation
- 2 states considered: powder and solid.

### Stakes at a macroscopic scale:

- **thermomechanics**: thermal expansion, residual stresses, solidification of the layer,
- **kinematics**: minimal execution time, easy to create.

# 3D model

**Physical characteristics:**  $A(t, x) = A_{sol.} \mathbb{1}_{sol.}(t, x) + A_{pou.} (1 - \mathbb{1}_{sol.}(t, x))$ .

**Constraints to satisfy:**

- change of state:  $\forall x \in D, \exists t$  tel que  $T(t, x) > T_\Phi$ ,
- thermal expansion:  $\forall x \in D, \forall t, \quad T(t, x) < T_M$ ,
- residual stresses.

**Simplifying the source model:**  $Q(t, x) = P \exp(-\delta|x - u(t)|^2)$ , with  $u(t)$  the laser path.

**Hypothesis :**

- no volumetric heat sources, no convection and no radiation.

**Heat equation:**

$$\left\{ \begin{array}{ll} \rho c_p \partial_t T - \nabla \cdot (\lambda \nabla T) = 0, & (t, x) \in (0, t_F) \times D, \\ \lambda \nabla T \cdot n = Q, & (t, x) \in (0, t_F) \times \partial D_{top}, \\ \lambda \nabla T \cdot n = 0, & (t, x) \in (0, t_F) \times \partial D_{side}, \\ T = T_{init}, & (t, x) \in (0, t_F) \times \partial D_{bot.}, \\ T(0, x) = T_{init}(x) & x \in D. \end{array} \right.$$

# 2D model: averaging in the manufacturing direction, orthogonal to the layer

## Hypothesis :

- adding a new diffusion term, characterized by  $\frac{\lambda_{sol}}{\Delta Z}$  with  $\Delta Z$  a characteristic length, to model the conduction on the vertical axis.

## Heat equation:

$$\begin{cases} \rho c_p \partial_t T - \nabla \cdot (\lambda \nabla T) + \frac{\lambda_{sol}}{\Delta Z} T = \frac{Q}{L}, & (t, x) \in (0, t_F) \times \Sigma, \\ \lambda \nabla T \cdot n = 0, & (t, x) \in (0, t_F) \times \partial \Sigma, \\ T(0, x) = T_{init}(x) & x \in \Sigma. \end{cases}$$

## Steady problem

# Optimization problem to consider

**Steady model:** the source is applied in the whole trajectory at the same time (heating thread).

**Objective :** vary the path  $\Gamma$  in order to minimize its length ( $J(\Gamma)$ ) while satisfying the change of phase ( $C_\Phi$ ) and maximal temperature constraints ( $C_M$ ).

$$\min_{\Gamma} J(\Gamma) = \int_{\Gamma} ds$$

while satisfying the constraints

$$\begin{cases} C_\Phi = \int_{\Sigma} [(T_\Phi - T)^+]^2 dx = 0 & (T > T_\Phi), \\ C_M = \int_{\Sigma} [(T - T_M)^+]^2 dx = 0 & (T < T_M). \end{cases}$$

and  $T$  solution of:

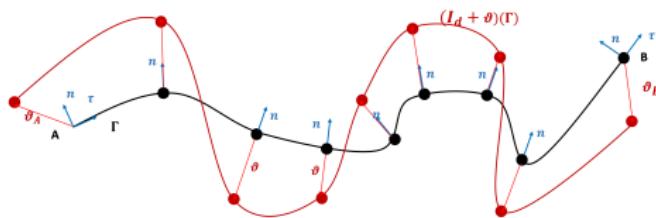
$$\begin{cases} -\nabla \cdot (\lambda_{pou.} \nabla T) + \frac{\lambda_{sol.}}{L \Delta Z} T = \frac{P}{L} \chi_{\Gamma}, & (t, x) \in (0, t_F) \times \Sigma \\ \nabla T \cdot n = 0 & (t, x) \in (0, t_F) \times \partial \Sigma \end{cases}$$



# Computation of the speed of variation: shape optimization (Allaire, Jouve, and Toader [2] and Henrot and Pierre [4])

**Shape optimization:** variation with respect to a vector field  $\vartheta$ ,

$\Gamma$  regular curve with chosen orientation, which tangent is  $\tau$ , which normal is  $n$  and curvature  $\kappa$  with  $A$  and  $B$  its endpoints.



- Shape derivative of  $J(\Gamma) = \int_{\Gamma} f(s) ds$ :

$$J'(\Gamma)(\vartheta) = \int_{\Gamma} [\partial_n f + \kappa f] \vartheta \cdot n ds + f(B)\vartheta(B) \cdot \tau(B) - f(A)\vartheta(A) \cdot \tau(A)$$

- Then:

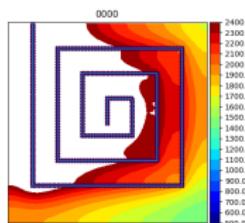
$$J(\Gamma^{n+1}) = J(\Gamma^n) + J'(\Gamma^n)(\vartheta) + o(\vartheta)$$

and  $\vartheta$  is chosen such that  $J(\Gamma^{n+1}) \leq J(\Gamma^n)$ .

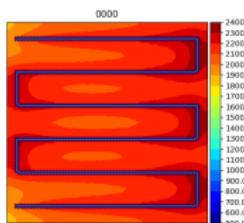
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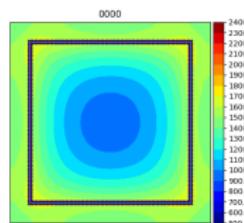
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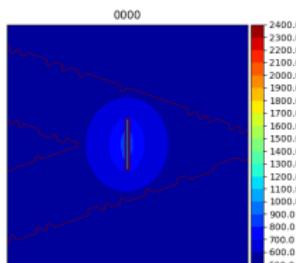
$$\alpha_{reg} = 10$$



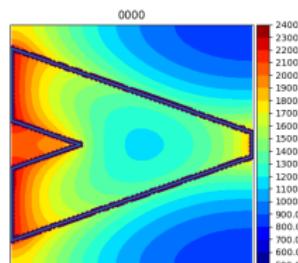
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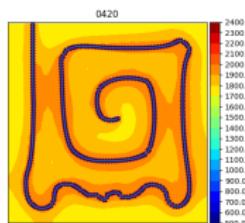
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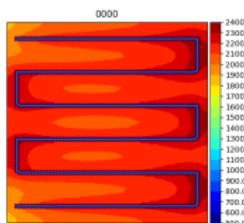
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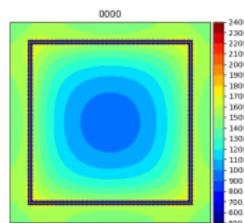
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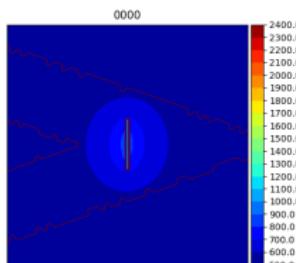
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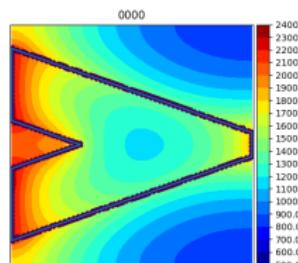
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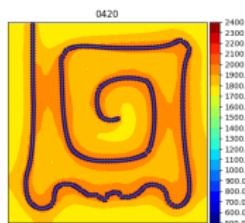
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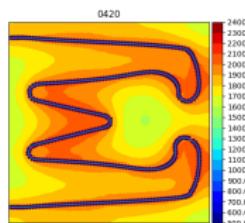
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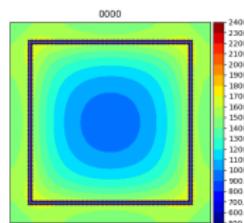
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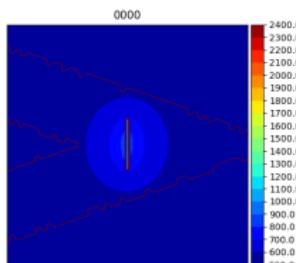
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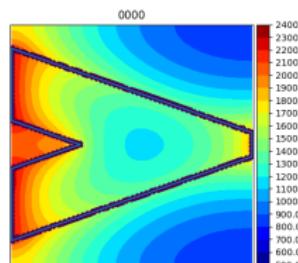
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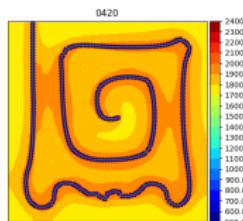
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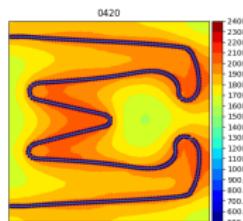
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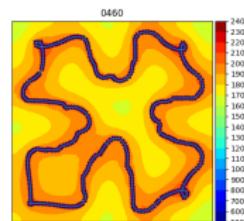
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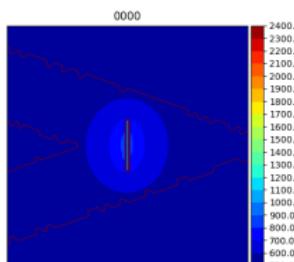
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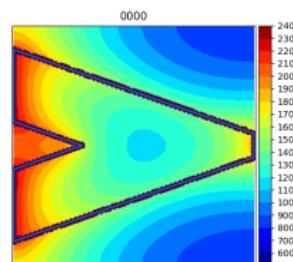
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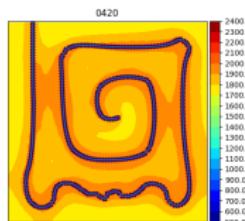
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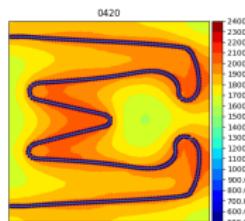
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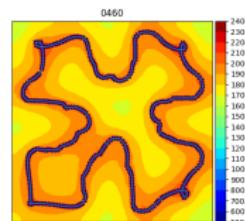
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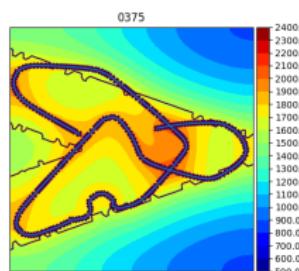
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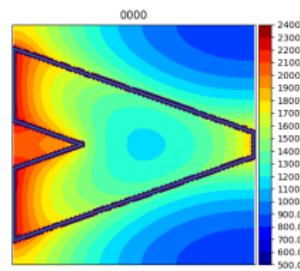
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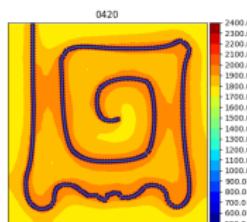
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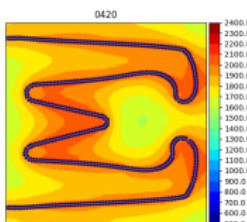
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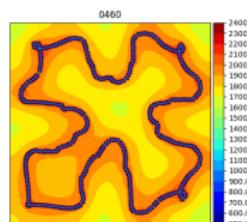
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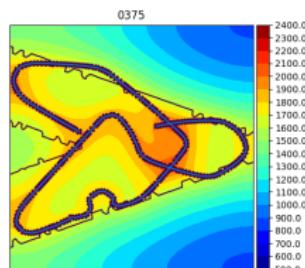
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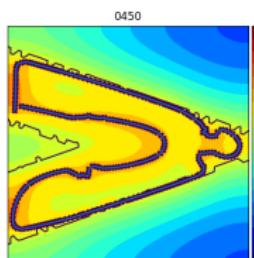
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## Unsteady problem

# Optimization problem to consider

**Unsteady model:** the source moves along the path.

**Objective :** manufacture the item by optimizing a fixed length path, with a source moving along at constant speed (velocity  $V > 0$  fixed and final time  $t_F > 0$  fixed).

$$\min_{\Gamma} J(\theta) = \int_D \left[ \left( T_\Phi - \max_t (|T(.,x)|) \right)^+ \right]^2 dx \approx \int_D \left[ (T_\Phi - \|T(.,x)\|_{L^p(0,t_F)})^+ \right]^2 dx.$$

with  $T$  solution of:

$$\begin{cases} \rho c_p \partial_t T - \nabla \cdot (\lambda \nabla T) + \frac{\lambda_{sol}}{L \Delta Z} T = \frac{Q(t,x)}{L}, & (t,x) \in (0, t_F) \times \Sigma, \\ \lambda \nabla T \cdot n = 0, & (t,x) \in (0, t_F) \times \partial \Sigma, \\ T(0, x) = T_{init}(x) & x \in \Sigma. \end{cases}$$

# Optimal control of the line (Lions [5] and Wendl, Pesch, and Rund [10])

**Control of the line:** using an angle  $\theta$ , formed by the horizontal vector and the tangent at each point.

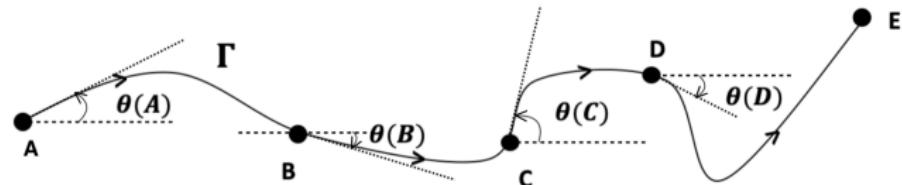
$$\min_{\Gamma} J(\theta)$$

while satisfying:

$$\begin{cases} \rho c_p \partial_t T - \nabla \cdot (\lambda \nabla T) + \frac{\lambda_{sol}}{L \Delta Z} T = \frac{Q(t,x)}{L}, & (t, x) \in (0, t_F) \times \Sigma, \\ \lambda \nabla T \cdot n = 0, & (t, x) \in (0, t_F) \times \partial \Sigma, \\ T(0, x) = T_{init}(x) & x \in \Sigma. \end{cases}$$

with  $Q(t, x) = P \exp(-\delta|x - u(t)|^2)$ , where the path equation  $u$  is given by:

$$\begin{cases} \dot{u}(t) = VF(\theta(t)) = V(\cos(\theta(t)), \sin(\theta(t))), & \forall t \in (0, t_F) \\ u(0) = u_0 \end{cases}$$



# Optimal control of the line direction, solving strategy

Introduction of a Lagrangian function, with two adjoints:

$$\mathcal{L}(\theta, T, p, u, w) = J(\theta)$$

$$\begin{aligned}
 & + \int_0^{t_F} \int_{\Sigma} \left[ \left( \rho c_p \partial_t T + \frac{\lambda_{sol}}{L \Delta Z} T - \frac{Q}{L} \right) p + \lambda \nabla T \nabla p \right] dx dt + \int_{\Sigma} \rho c_p (T(0, x) - T_{init}) p(0, x) dx \\
 & + \int_0^{t_F} [(\dot{u}(t) - VF(\theta(t))) w] dt + (u(0) - u_0) w(0).
 \end{aligned}$$

If  $u$  the path and  $T$  the temperature, for any adjoints  $p$  and  $w$ ,  $\mathcal{L}(\theta, T, p, u, w) = J(\theta)$ , thus :

$$\begin{aligned}
 \frac{dJ}{d\theta}(\theta)(\tilde{\theta}) &= \frac{d\mathcal{L}}{d\theta}(\theta, T, p, u, w)(\tilde{\theta}) \\
 &= \partial_{\theta} \mathcal{L}(\theta, T, p, u, w)(\tilde{\theta}) \\
 &+ < \partial_u \mathcal{L}(\theta, T, p, u, w), \partial_{\theta} u(\theta)(\tilde{\theta}) > + < \partial_T \mathcal{L}(\theta, T, p, u, w), \partial_{\theta} T(\theta)(\tilde{\theta}) >
 \end{aligned} \tag{1}$$

giving the equations of  $p$  and  $q$ :

- $p^*$  related to heat equation ( $\partial_T \mathcal{L}(\theta, T, p^*, u, w) = 0$ ),
- $w^*$  related to the path equation ( $\partial_u \mathcal{L}(\theta, T, p^*, u, w^*) = 0$ ).



# Results:

**Values:** (Allaire and Jakabčin [1] and Van Belle [9])

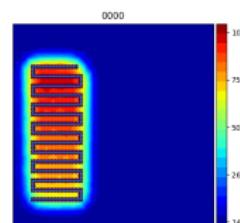
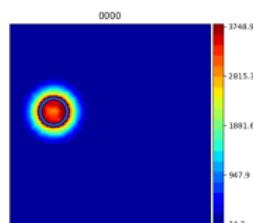
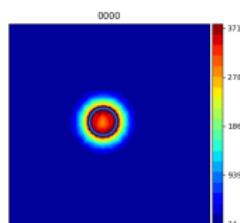
$$\lambda_{sol.} = 15 W.m^{-1}K^{-1}, \quad \lambda_{pou.} = 0.25 W.m^{-1}K^{-1},$$

$$\rho_{sol.} = 8000 kg.m^{-3}, \quad \rho_{pou.} = 4000 kg.m^{-3},$$

$$c_{sol.} = c_{pou.} = 450 J.kg^{-1}.K^{-1}$$

$$L = 10 cm, \quad \Delta Z = 10 cm, \quad P = 768000 * (10^4) W.m^{-2},$$

$$T_\Phi = 500 K, \quad T_{init} = 30 K., \quad p = 2.$$



$$\alpha_{reg} = 1, \quad |\Gamma| = 9 cm$$

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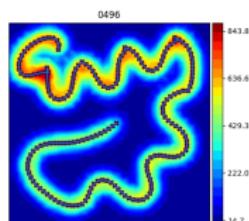
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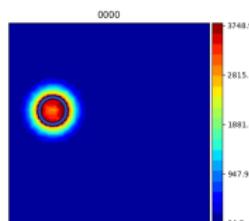
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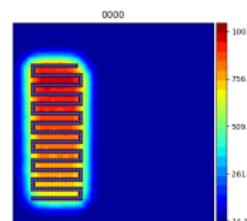
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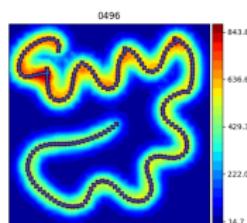
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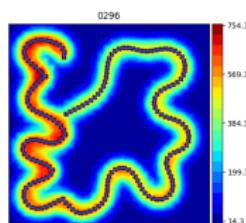
$$c_{sol.} = c_{pou.} = 450J.kg^{-1}.K^{-1}$$

$$L = 10cm, \quad \Delta Z = 10cm, \quad P = 768000 * (10^4) W.m^{-2},$$

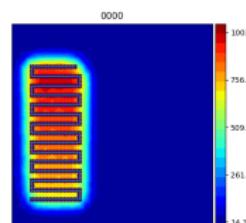
$$T_\Phi = 500K, \quad T_{init} = 30K., \quad p = 2.$$



$$\alpha_{reg} = 1, \quad |\Gamma| = 9cm$$



$$\alpha_{reg} = 1, \quad |\Gamma| = 9cm$$



$$\alpha_{reg} = 1, \quad |\Gamma| = 8.9cm$$

# Results:

**Values:** (Allaire and Jakabčin [1] and Van Belle [9])

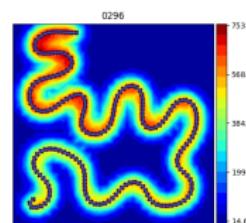
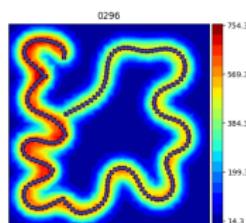
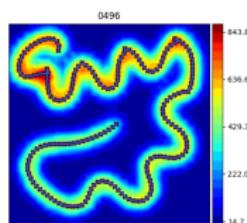
$$\lambda_{sol.} = 15W.m^{-1}K^{-1}, \quad \lambda_{pou.} = 0.25W.m^{-1}K^{-1},$$

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$$c_{sol.} = c_{pou.} = 450J.kg^{-1}.K^{-1}$$

$$L = 10cm, \quad \Delta Z = 10cm, \quad P = 768000 * (10^4) W.m^{-2},$$

$$T_\Phi = 500K, \quad T_{init} = 30K., \quad p = 2.$$



$$\alpha_{reg} = 1, \quad |\Gamma| = 9cm$$

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$$\alpha_{reg} = 1, \quad |\Gamma| = 8.9cm$$

## Conclusion and perspectives



# Perspectives

## Short terms perspectives:

- steady case:
  - adding more realistic constraints (geometrical, thermal et mechanical),
  - allow for the splitting of the path and gathering many,
  - adapt the curve meshing to the industrial requirements,
  - make the 2D case evolve to a layer by layer 3D optimization.
- unsteady case:
  - optimize with respect to the final time as well as the path,
  - improve the model,
  - perspectives from the steady case.

## Long term perspectives:

- coupling shape optimization to path optimization,
- optimize a line in 3D.

# Références I



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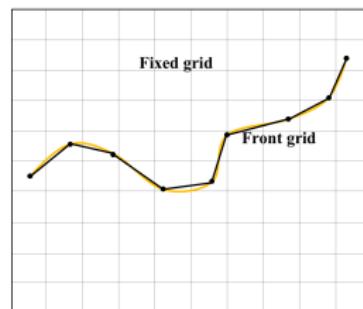
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# Numerical adaptation, steady problem

## Line modelling: front tracking methods (Tryggvason et al. [8])

Path discretization without modifying the mesh used for heat equation.

- control the curve's discretization and approximate continuous values (normal, curvature, ...),
- communicate between the discretization and the mesh.



## Algorithm

1. initial guess,
2. computation of the objective and constraints functions (heat equation),
3. computation of the shape derivative,
4. advection of the path and control of its discretization,
5. computation of the objective and constraints functions (heat equation),
- 6. if improvement: iteration accepted back to 3.**
- 7. else: iteration refused** the step is decreased and back to 4.