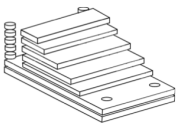


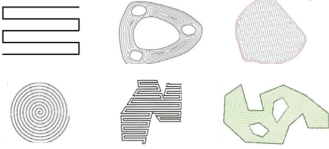
CONTEXT

Decomposition in layers



G. Allaire et al., Shape optimization of a layer-by-layer mechanical constraint for additive manufacturing, C. R. Math. Acad. Sci. Paris 355 (2017) 699-717

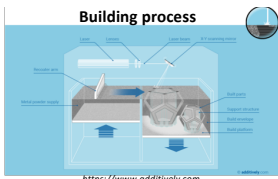
Choice of building parameters : laser path, speed, power



Evaluation of the process

drawbacks in the item's quality (thermal expansion, residual stresses, ...)

evaluation of the process's speed (kinematics)



<https://www.additively.com>

2D MODEL

Physical characteristics : $A(t, x) = A_{sol}1_{sol}(t, x) + A_{pow}(1 - 1_{sol}(t, x))$

Constraints to satisfy :

- change of state $\forall x \in \Sigma, \exists t \text{ tel que } T(t, x) > T_\Phi$
- thermal expansion $\forall x \in \Sigma, \forall t, T(t, x) < T_M$

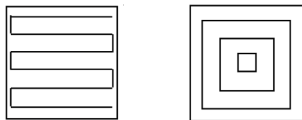
Source : $Q(t, x) = P e^{-\delta(x-u(t))^2}$

Heat Equation :

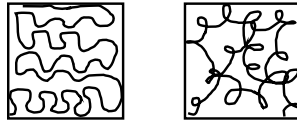
$$\begin{cases} \rho c_p \partial_t T - \nabla \cdot (\lambda \nabla T) + \frac{\lambda_{sol}}{L \Delta Z} T = \frac{Q}{L} & (t, x) \in (0, t_f) \times \Sigma, \\ \lambda \nabla T \cdot n = 0, & (t, x) \in (0, t_f) \times \partial \Sigma \\ T(0, x) = T_{init}(x), & x \in \Sigma \end{cases}$$

OBJECTIVE

Existing laser paths based on predefined patterns.



Why are these patterns chosen? Are they optimal? Why not these?



Optimize the laser path to improve both the mechanical and kinematic properties of the process, without basing it on any pattern.

STEADY CASE (source applied on the whole path : heating thread)

Optimization problem

$$\min_{\Gamma} J(\Gamma) = \int_{\Gamma} ds$$

under the constraints :

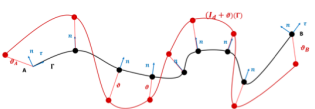
$$C_\Phi(\Gamma) = \int_{\Sigma} [(T_\Phi - T)^+]^2 dx = 0 \quad (T > T_\Phi),$$

$$C_M(\Gamma) = \int_{\Sigma} [(T - T_M)^+]^2 dx = 0 \quad (T < T_M).$$

and T solution of :

$$\begin{cases} -\nabla \cdot (\lambda_{pow} \nabla T) + \frac{\lambda_{sol}}{L \Delta Z} T = \frac{P}{L} 1_{\Gamma}, & x \in \Sigma, \\ \lambda_{pow} \nabla T \cdot n = 0 & x \in \partial \Sigma. \end{cases}$$

Shape optimization



$$J(\Gamma) = \int_{\Gamma} f(s) ds$$

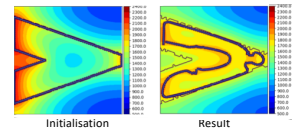
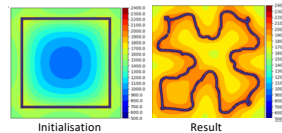
$$J'(\Gamma)(\vartheta) = \int_{\Gamma} [\partial_n f + \kappa f] \vartheta \cdot n ds + f(B)\vartheta(B) \cdot \tau(B) - f(A)\vartheta(A) \cdot \tau(A)$$

Results

(Values from L. Van Belle, Analyse, Modélisation et Simulation de l'apparition de contraintes en Fusion Laser Métallique, Lyon, INSA, Nov. 2013)

Fusion of the whole rectangle: $1700 < T < 2000$

Fusion of a cantilever:



Future work

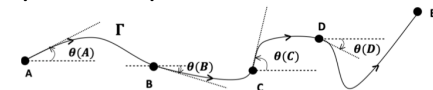
- Adding constraints
- Allow for splitting a path and gathering paths
- 3D optimization: consider the evolution of the path depending on the layer

UNSTEADY CASE (source moving along the path)

Assumptions

- fixed velocity V (constant along the path)
- fixed power P (constant along the path)
- final time fixed t_f (length of the path fixed)

Optimal control problem

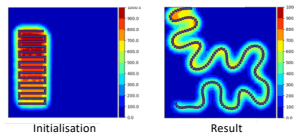
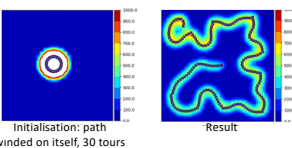


$$\min_{\Gamma} J(\vartheta) = \int_{\Sigma} [(T_\Phi - \max_x (|T(\cdot, x)|))]^2 dx \approx \int_{\Sigma} [(T_\Phi - |T(\cdot, x)|)_{L^2(0, t_f)}]^2 dx$$

with T solution of the heat equation, where $Q(t, x) = P e^{-\delta(x-u(t))^2}$ and

$$\begin{cases} \dot{u}(t) = F(\vartheta(t)) = (\cos(\vartheta(t)), \sin(\vartheta(t))) & t \in (0, t_f) \\ u(0) = u_0 \end{cases}$$

Results Objective: $500 < T$



Future work

- Optimizing with respect to the final time
- Same work than for the steady case

PERSPECTIVES

- coupling the shape and path optimization
- optimizing a line in a 3D domain

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- M. Megahed et al. "Metal Additive-Manufacturing Process and Residual Stress Modeling", Integrating Materials and Manufacturing Innovation 5.1 (Dec. 2016), p. 4.
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- J. L. Lions. Optimal Control of Systems Governed by Partial Differential Equations. Vol. 170. Springer (1971).
- S. Wendl, H. J. Pesch, and A. Rund. "On a State-Constrained PDE Optimal Control Problem Arising from ODE-PDE Optimal Control", Recent Advances in Optimization and Its Applications in Engineering. (2010), pp. 429-438.